SyMan Lesson 7: Completing the Square

In previous exercises, you have learned to solve quadratic equations through a combination of graphing and factoring. This exercise shows how to solve quadratics that are difficult to solve with those techniques.

As you follow along with the example below, be sure to read the explanation after each step. These explanations tell you why you are doing each step, and give further helpful advice.

Step 1 **Try to use the graphing/factoring technique to factor "`49x\s(2)+42x+8`". What is the problem with factoring this trinomial?**

Step 2 **Enter the equation "`49x\s(2)+42x+8 = 0`".**

It is possible to solve for "`x`", even though the roots of this equation are not integers, or even reasonable rationals like `\f(1,2)` or `\f(1,4)`. The technique to use for a question like this is called "Completing the Square".

Step 3 **Eliminate the coefficient on the "`x\s(2)`" term by dividing by `49`, expanding, and simplifying.**

It is usually easier to factor trinomials with a coefficient of `1` on the "`x`" term, so it is reasonable to perform this step.

Step 4 **Eliminate the numeric term, `\f(8,49)` , from the left side (it will, of course, end up on the right side with a changed sign).**

The trinomial can be written in the form of a perfect square, but the numeric term must be temporarily moved over to the right side.

Step 5 **Add "`\b(\f(3,7))\s(2)`" and then simplify the equation. (Remember, this is entered as `(3/7)^2`)**

Recall that a perfect square is an expression like `(x+4)\s(2) = x\s(2)+8x+16`. Notice that the final numeric term, `16`, is the square of half the "`x`" coefficient, `8`— i.e. `(8/2) = 4`, and `4\s(2) = 16`.

For our equation, the left side,."`x\s(2) + \f(6,7) x`", would be a perfect square if there were a numeric term of `\b(\f(3,7))\s(2)` (note that one-half of `\f(6,7)` is `\f(3,7)` , and for a perfect square, we need the square of this amount).

Step 6 **Select the left side of the equation by clicking on the x\s(2) term and dragging across the entire left side. Hold the mouse button down until the entire left side is hilighted in black, then release the button.**

In order to factor the left side only, we need to select it first. If part of an expression or equation is selected like this, SyMan will operate on the selection only, rather than on the entire active equation.

Step 7 **Since the left side is a perfect square of (x + \f(3,7)), tell SyMan to factor this expression and simplify the result.**

The active equation should become (x + \f(3,7))\s(2) = \f(1,49) .

Step 8 **Click on the "√" button in the upper-left corner to take the square root of both sides of the equation. Simplify the result.**

Essentially, the equation is in the form "a\s(2) = n", an equation which, when you take the square root of both sides, gives you "a = ±\r(n)". In this case, you should end up with x + \f(3,7) = \f(1,7) .

Step 9 **Due to the ±\r(\f(1,49)), there are actually two correct solution to this quadratic equation: x+\f(3,7) = \f(1,7) and x+\f(3,7) = \f(-1,7). In the space below, write the values for x that satisfy these solutions.**

x = ________________________ or x = ________________________

Unfortunately, SyMan considers only the positive root when it simplifies an expression like \r(\f(1,49)) You have to remember that the negative root is also a solution.

Step 10 **Graph the equation y = 49x\s(2)+42x+8 and use the graph to find the approximate roots of the equation. Write these roots below:**

from the graph, x = ____________________ or x = ____________________

Step 11 **Compare the results from the Step 10 to the results from step 9. They should be nearly identical, though the algebraically-determined results are more correct (remember, the graph gives only approximate answers).**

Completing the square to factor and solve an expression is more difficult than most other factoring techniques; however, it works for problems which would otherwise be impossible to solve exactly. For example, we saw that 49x\s(2)+42x+8 could not be solved graphically because the roots were neither integer, nor easily-recognizable fractions. Yet, by completing the square, we found that the exact roots were \f(-2,7) and \f(-4,7)

In general, the steps to factoring by completing the square are:

49x\s(2) + 42x + 8 = 0

i) divide by the coefficient of the squared term — x\s(2) + \f(42,49) x + \f(8,49) = 0

ii) eliminate the numeric term from the left-hand side (note: \f(42,49) = \f(6,7)) — x\s(2) + \f(6,7) x = \f(-8,49)

iii) add the square of one-half of the coefficient of x — x\s(2) + \f(6,7) x + (\f(3,7))\s(2) = \f(-8,49) + (\f(3,7))\s(2)

iv) simplify — x\s(2) + \f(6,7) x + \f(9,49) = \f(1,49)

v) factor the perfect square on the lefthand side — (x + \f(3,7))\s(2) = \f(1,49)

vi) take the square root of both sides — \r((x + \f(3,7))\s(2)) = \r(\f(1,49))

vii) simplify — x + \f(3,7) = ± \f(1,7)

viii) solve for the TWO values of x — x + \f(3,7) = \f(1,7) **or** x + \f(3,7) = \f(-1,7)

ix) write your final answers — x = \f(-2,7) **or** x = \f(-4,7)

When you have completed Steps 1 through 11, go on to answer the following questions:

1.) Solve the following expressions by completing the square. Follow along with the steps of the example until you feel comfortable working on your own. Watch for negatives!

a) 9x\s(2) + 6x – 1 = 0 x = _________ or x = _________

b) 4x\s(2) + 4x – 8 = 0 x = _________ or x = _________

c) 25x\s(2) – 10x – 15 = 0 x = _________ or x = _________

d) 2x\s(2) + 6x + 3 = 0 x = _________ or x = _________

e) 7x\s(2) – 14x + 2 = 0 x = _________ or x = _________

f) 147x\s(2) + 42x – 95 = 0 x = _________ or x = _________

g) 9x\s(2) + 12x + 4 = 0 x = _________ or x = _________

2.) Try factoring the following by completing the square. Treat a, b, c as constants.

a) ax\s(2) + bx + c = 0

Solutions: x =

4.) What do you suppose happens if you try to use the same technique to factor equations which have no real roots? Describe the problem with factoring 3x\s(2) – 9x + 15 = 0, which has no real roots.